HW2A. Written Homework 2A. **Due Week 2 Wednesday 11:59PM**

Name: Answer Key

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

(1) For each p-series given below, determine p and determine if the series is convergent or divergent.

$\sum \frac{1}{n^p}$ is conv. $\Leftrightarrow p > 1.$

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $p=2$; convergent

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ $p=\frac{3}{2}$; convergent

(c) $\sum_{n=1}^{\infty} n^2 = \sum_{n=1}^{\infty} \frac{1}{n^{-2}}$; divergent

(d) $\sum_{n=1}^{\infty} n^{-\frac{1}{2}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$; $p=\frac{1}{2}$; divergent

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{4}}}$; $p=\frac{3}{4}$; divergent

(2) For each geometric series given below, determine the common ratio r (i.e. r such that $a_{n+1} = r \cdot a_n$ with (a_n) the terms of the series) and determine if the series is convergent or divergent.

$\sum ar^n$ conv. $\Leftrightarrow |r| < 1.$

(a) $\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$ $r=\frac{1}{5}$; convergent.

(b) $\sum_{n=2}^{\infty} \left(\frac{11}{7}\right)^n$ $r=\frac{11}{7}$; divergent.

(c) $\sum_{n=3}^{\infty} \frac{(5)(-7)^{n+1}}{3^n} = \sum_{n=3}^{\infty} (-35)\left(-\frac{7}{3}\right)^n$; $r=-\frac{7}{3}$; divergent

(d) $\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$; $r=\frac{1}{2}$; convergent

(e) $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^{-n} = \sum_{n=0}^{\infty} \left(-\frac{3}{2}\right)^n$; $r=-\frac{3}{2}$; divergent.

(f) $\sum_{n=0}^{\infty} \frac{3(-1)^{n+1}}{4^n} = \sum_{n=0}^{\infty} (-3)\left(-\frac{1}{4}\right)^n$; $r=-\frac{1}{4}$; convergent.

(3) Use the **Divergence Test** to determine the convergence of each of the following series. If the divergence test is inconclusive, write "inconclusive". Note that while there may be other ways to determine the convergence of the series below, you are only expected to use the Divergence Test.

(a) $\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$ $\lim_{n\to\infty} \left(\frac{1}{5}\right)^n = 0$; DT is inconclusive.

(b) $\sum_{n=1}^{\infty} \left(\frac{n^2+n+1}{n+2}\right)$ $\lim_{n\to\infty} \frac{n^2+n+1}{n+2} = \lim_{n\to\infty} \frac{2n+1}{1} = \infty \neq 0$; $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n+2}$ is divergent by DT.

(c) $\sum_{n=1}^{\infty} \left(\frac{n^4+2n^2+1}{e^n}\right)$ $\lim_{n\to\infty} \frac{n^4+2n^2+1}{e^n} = 0$; DT is inconclusive.

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n+1}{n^2+n+1}\right)$ Use the Squeeze Theorem: If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.
Since $\lim_{n\to\infty} \left(\frac{n+1}{n^2+n+1}\right) = 0$, $\lim_{n\to\infty} (-1)^{n+1} \left(\frac{n+1}{n^2+n+1}\right) = 0$. DT is inconclusive.

(e) $\sum_{n=1}^{\infty} \frac{1}{n!}$ $\lim_{n\to\infty} \frac{1}{n!} = 0$ since $\lim_{n\to\infty} n! = \infty$; DT is inconclusive;

(f) $\sum_{n=1}^{\infty} \frac{a^n}{n!}$ for any $a \in \mathbb{R}$ $\lim_{n\to\infty} \frac{a^n}{n!} = 0$; DT is inconclusive.
This is supposed to be given to you.